

An alarm clock has a randomized snooze feature. When it rings, if you hit the snooze button to silence it, the alarm waits a random number of minutes (X), then rings again. The probability density function for X is SCORE: ____ / 30 PTS

$$f(x) = \begin{cases} k \sqrt[3]{x}, & x \in [1, 8] \\ 0, & x \notin [1, 8] \end{cases} \text{ (for some appropriate constant } k \text{).}$$

[a] Find the probability that the alarm rings again less than 4 minutes after you hit the snooze button.

Your final answer must be a number, not an integral.

$$\int_1^8 k x^{\frac{1}{3}} dx = 1$$

$$k \cdot \frac{3}{4} x^{\frac{4}{3}} \Big|_1^8 = 1$$

$$k \cdot \frac{3}{4} (16 - 1) = 1$$

$$k = \frac{4}{45} \text{ (3)}$$

$$\int_1^4 \frac{4}{45} x^{\frac{1}{3}} dx = \frac{4}{45} \cdot \frac{3}{4} x^{\frac{4}{3}} \Big|_1^4$$

$$= \frac{1}{15} (4^{\frac{4}{3}} - 1) \text{ (3)}$$

(4) POINTS EACH

EXCEPT AS INDICATED

[b] Find the median number of minutes before the alarm rings again.

$$\int_1^M \frac{4}{45} x^{\frac{1}{3}} dx = \frac{1}{2}$$

$$\frac{4}{45} \cdot \frac{3}{4} x^{\frac{4}{3}} \Big|_1^M = \frac{1}{2}$$

$$M^{\frac{4}{3}} - 1 = \frac{15}{2}$$

$$M = \left(\frac{17}{2}\right)^{\frac{3}{4}} \text{ MINUTES}$$

Find the value of c guaranteed by the Mean Value Theorem for Integrals for $f(x) = 3x^2 - 4x$ on $[-2, 1]$.

SCORE: ____ / 20 PTS

$$f_{\text{AVE}} = \left[\frac{\int_{-2}^1 (3x^2 - 4x) dx}{1 - (-2)} \right] \textcircled{5}$$

$$= \textcircled{4} \frac{(x^3 - 2x^2) \Big|_{-2}^1}{3}$$

$$= \frac{(1 - -8) - 2(1 - 4)}{3}$$

$$= \underline{5} \textcircled{3}$$

$$f(c) = \underline{3c^2 - 4c = 5} \textcircled{3}$$

$$3c^2 - 4c - 5 = 0$$

$$c = \frac{4 \pm \sqrt{16 + 60}}{6}$$

$$= \frac{4 \pm \sqrt{76}}{6}$$

$$= \underline{\frac{2 \pm \sqrt{19}}{3}} \textcircled{3}$$

$$c = \underline{\frac{2 - \sqrt{19}}{3}} \textcircled{2}$$

$$\frac{2 + \sqrt{19}}{3} > \frac{2 + 4}{3}$$

$$= 2$$

Find the area of the surface created by revolving the arc of $f(x) = \frac{2x^6 + 1}{8x^2}$ on $[1, 2]$ about the y -axis.

SCORE: ____ / 25 PTS

Your final answer must be a number, not an integral.

$$= \frac{1}{4}x^4 + \frac{1}{8}x^{-2}$$

$$f'(x) = x^3 - \frac{1}{4}x^{-3}$$

$$\int_1^2 \underbrace{2\pi x}_{(2)} \underbrace{\sqrt{1 + (x^3 - \frac{1}{4}x^{-3})^2}}_{(3)} dx$$

$$= 2\pi \int_1^2 x \sqrt{1 + x^6 - \frac{1}{2} + \frac{1}{16}x^{-6}} dx$$

$$= 2\pi \int_1^2 x \sqrt{x^6 + \frac{1}{2} + \frac{1}{16}x^{-6}} dx$$

$$= 2\pi \int_1^2 x \underbrace{(x^3 + \frac{1}{4}x^{-3})}_{(4)} dx$$

$$= 2\pi \int_1^2 (x^4 + \frac{1}{4}x^{-2}) dx$$

$$= 2\pi \left(\frac{1}{5}x^5 - \frac{1}{4}x^{-1} \right) \Big|_1^2$$

$$= 2\pi \left(\frac{1}{5}(32-1) - \frac{1}{4}(\frac{1}{2}-1) \right)$$

$$= 2\pi \left(\frac{31}{5} + \frac{1}{8} \right)$$

$$= 2\pi \cdot \frac{253}{40}$$

$$= \frac{253\pi}{20} \quad (3)$$

Find the area between the curves $y = 3x^2 - 6x$ and $y = 2x + 3$ over the interval $x \in [1, 4]$.

SCORE: ____ / 25 PTS

Your final answer must be a number, not an integral. HINT: The answer is NOT 6.

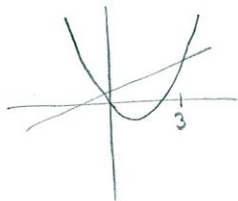
$$3x^2 - 6x = 2x + 3$$

$$3x^2 - 8x - 3 = 0 \quad (3)$$

$$(3x+1)(x-3) = 0$$

$$x = -\frac{1}{3}, 3 \quad (3)$$

$x \in [1, 4]$



$$\begin{aligned} & \textcircled{2} \int_1^3 (2x+3 - (3x^2-6x)) dx + \int_3^4 (3x^2-6x - (2x+3)) dx \quad \textcircled{2} \\ & = \int_1^3 (-3x^2 + 8x + 3) dx + \int_3^4 (3x^2 - 8x - 3) dx \quad \textcircled{3} \quad \textcircled{3} \\ & = (-x^3 + 4x^2 + 3x) \Big|_1^3 + (x^3 - 4x^2 - 3x) \Big|_3^4 \quad \textcircled{3} \\ & = -(27-1) + 4(9-1) + 3(3-1) \quad \textcircled{6} \\ & \quad + (64-27) - 4(16-9) - 3(4-3) \\ & = 11 + 4 + 3 \\ & = 18 \quad \textcircled{3} \end{aligned}$$

A solid of revolution has volume $2\pi \int_2^6 (y+6)(\sqrt{2y} - \frac{y}{2})^2 dy$.

SCORE: ____ / 15 PTS

Find the equation of the axis of revolution, and the equations of the boundaries of the region being revolved.

Sketch & shade in the region being revolved.

Equation of axis of revolution:

$\frac{1}{2}$ $y = -6$

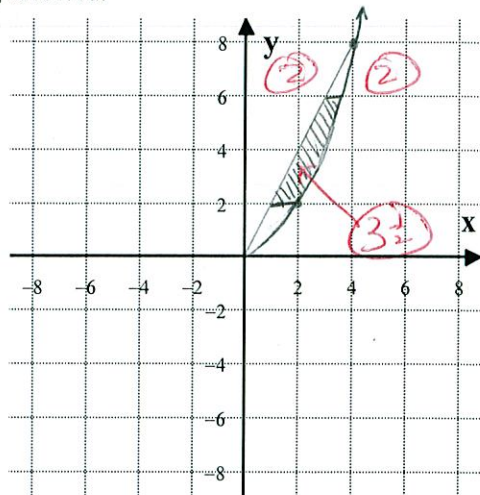
Equations of boundaries:

$\frac{1}{2}$ $y = 2$

$\frac{1}{2}$ $y = 6$

$\frac{1}{2}$ $x = \sqrt{2y}$ or $y = \frac{1}{2}x^2$

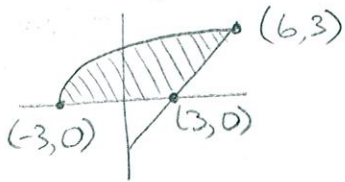
$\frac{1}{2}$ $x = \frac{y}{2}$ or $y = 2x$



Consider the region defined by $y \leq \sqrt{x+3}$, $y \geq x-3$ and $y \geq 0$.

SCORE: ____ / 35 PTS

- [a] Write, **BUT DO NOT EVALUATE**, a single integral for the volume of the resulting solid if the region is revolved around the line $x = 10$.



$$\sqrt{x+3} = x-3$$

$$x+3 = x^2-6x+9$$

$$0 = x^2-7x+6 \quad (3)$$

$$= (x-1)(x-6)$$

$$x = 1 \text{ or } 6$$

$$\downarrow$$

$$y = -2 \text{ or } 3 \quad (2)$$

SLICE HORIZONTALLY FOR 1 INTEGRAL
 AXIS OF REVOLUTION VERTICAL
 WASHER METHOD

$$y = \sqrt{x+3} \rightarrow x = y^2-3$$

$$y = x-3 \rightarrow x = y+3 \quad (4)$$

$$\pi \int_0^3 [(10-(y^2-3))^2 - (10-(y+3))^2] dy$$

(2) (4) (3) (3)

- [b] Suppose the region is the base of a solid. Cross sections of the solid perpendicular to the x -axis are semicircles. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\frac{\pi}{8} \left[\int_{-3}^3 (\sqrt{x+3} - 0)^2 dx + \int_3^6 (\sqrt{x+3} - (x-3))^2 dx \right]$$

(3) (3) (2) (3) (3)